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Powered Swingby

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The problem of applying an impulsive thrust in a spacecraft that is performing a swingby maneuver (also called gravity-assisted maneuver) is studied. The objective is to derive a set of analytical equations that can calculate the change in velocity, energy, and angular momentum for this maneuver as a function of the three usual parameters of the standard swingby maneuver plus the two parameters (the magnitude of the impulse and the angle between the impulse and the velocity of the spacecraft) that specify the planar impulse applied. The dynamics used to obtain those equations are given by the patched-conic approach. A study is also performed to find in which cases the impulse is more efficient when applied during that close approach or after that, in a two-step maneuver. After that, the same maneuvers are computed under the dynamics given by the restricted three-body problem, and the results are compared with the ones obtained previously under the patched-conic dynamics.

Introduction

THE swingby maneuver is a very popular technique used to decrease the fuel expenditure in space missions. The standard maneuver uses a close approach with a celestial body to modify the velocity, energy, and angular momentum of the spacecraft. There are many important applications, such as Voyager I and II, that used successive close encounters with the giant planets to make a long journey to the outer solar system[1]; the Ulysses mission that used a close approach with Jupiter to change its orbital plane to observe the poles of the sun[2]; etc.

In this paper, a different type of swingby maneuver is studied, where we apply an impulse to the spacecraft during its closest approach with the celestial body. This type of maneuver increases the alternatives available to mission designers for meeting the requirements of many missions. This type of swingby has been studied before by several researchers. Two excellent papers available in the literature are the ones written by Gobetz[3] and Walton et al.[4] Gobetz[3] studied two- and three-dimensional optimum transfers between hyperbolic asymptotes with one or four impulses. The four-impulse maneuver has the advantage of reducing the total ΔV in some cases and it also increases the time that the spacecraft remains close to the celestial body involved in the maneuver. Walton et al.[4] generalized those results and solved the problem of the optimum powered swingby maneuver between two given hyperbolic excess velocity vectors around a real celestial body (a planet or a satellite). They also found multi-impulsive transfers in several cases. In the present paper, equations are derived to give us the change in velocity, energy, and angular momentum as a function of the three independent parameters (required to describe the standard swingby maneuver) described in the next section and the new two parameters that belong to this particular model: the magnitude of the impulse applied and the angle that this impulse makes with the velocity of the spacecraft. The maximum transfer of energy occurs for impulses applied in a direction that makes an angle of about 20 deg with the velocity of the spacecraft. But, since the maximum transfer of energy is not always the goal of the mission, we give the additional flexibility of allowing any direction that belongs to the plane of motion of the two primaries for the impulse. All of those equations are derived assuming that 1) the maneuver can be modeled by the patched-conic model (a series of Keplerian orbits), 2) the impulse is applied during the passage by the periapse, 3) the impulse changes the velocity of the spacecraft instantaneously, and 4) the motion is planar everywhere.

After that, this powered swingby is compared with a different maneuver, where the impulse is not applied during the close approach, but just after the spacecraft leaves the sphere of influence of the celestial body. In that way, the best position to apply an impulse in the spacecraft is investigated: during the close approach with the celestial body or after that, in a two-step maneuver, where the first step is the swingby and the second step is the impulse.

Those maneuvers are then recalculated, using the more realistic dynamics given by the restricted three-body problem, and the results are compared.

Standard Swingby Maneuver

The standard swingby maneuver consists of using a close encounter with a celestial body to change the velocity, energy, and angular momentum of a smaller body (a comet or a spacecraft). This standard maneuver can be identified by three independent parameters: 1) $V_{\text{inf}-}$, the magnitude of the velocity of the spacecraft when approaching the celestial body, or V_p, the magnitude of the velocity of the spacecraft at periapse (those quantities are equivalent); 2) r_p, the distance between the spacecraft and the celestial body during the closest approach; and 3) ψ, the angle of approach (angle between the periapse line and the line that connects the two primaries).

Figure 1 shows the sequence for this maneuver and some of those and other important variables.

It is assumed that the system has three bodies: a primary (M_1) and a secondary (M_2) body with finite mass that are in circular orbit around their common center of mass and a third body with negligible mass (the spacecraft) that has its motion governed by the two other bodies. We can see that the spacecraft leaves the point A, crosses the horizontal axis (the line between M_1 and M_2), passes by the point P (the periapsis of the trajectory of the spacecraft around M_2), and goes to the point B. We choose the points A and B in such

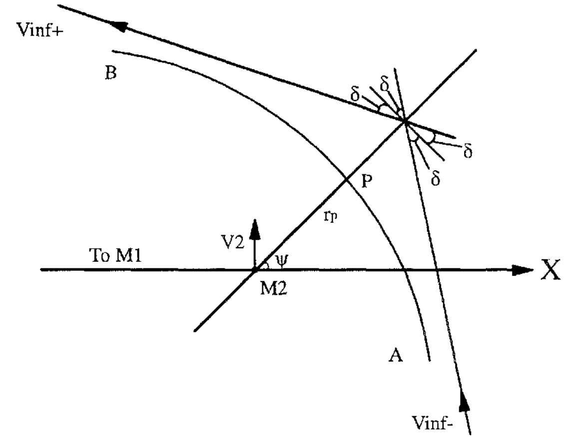

Fig. 1 Standard swingby maneuver.

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a way that we can neglect the influence of M_2 at those points and, consequently, we know that the energy is constant after B and before A (the system follows the two-body celestial mechanics). Two of our initial conditions are clearly identified in the figure: the perigee distance r_p (distance measured between the point P and the center of M_2) and the angle ψ, measured from the horizontal axis in the counterclockwise direction. The distance r_p is not to scale, to make the figure easier to understand. In this paper only the planar motion (all of the three bodies always in the same plane) is studied. The result of this maneuver is a change in velocity, energy, and angular momentum in the Keplerian orbit of the spacecraft around the central body. Using the patched-conic approximation, the equations that quantify those changes are available in the literature. Under this approximation the maneuver is considered as composed of three parts, where each of those systems is governed by the two-body celestial mechanics. The first system describes the motion of the spacecraft around the primary body before the close encounter (the secondary body is neglected). When the spacecraft comes close to the secondary body, the primary is neglected, and a second two-body system is formed by the spacecraft and the secondary body. After the close encounter, the spacecraft leaves the secondary body, and it goes to an orbit around the primary body again. Then the secondary is neglected one more time. One of the best descriptions of this maneuver and the derivation of the equations is in Broucke.[5] The equations are as follows:

$$\delta = \sin^{-1}\left[\frac{1}{1+\left(r_p V_{\text{inf}-}^2/\mu_2\right)}\right] \tag{1}$$

$$\Delta E = -2V_2 V_{\text{inf}-} \sin\delta \sin\psi \tag{2}$$

$$\Delta C = -2V_2 V_{\text{inf}-} \sin\delta \sin\psi \tag{3}$$

$$\Delta V = 2V_{\text{inf}-} \sin\delta \tag{4}$$

Those equations are valid for a system where the angular speed ω of the rotation of the two primaries is unity. Otherwise, Eq. (3) would be multiplied by ω. The presence of this unitary ω (whose unit is s^{-1}) explains the apparent conflict of units in Eqs. (2) and (3).

In those equations δ is half of the total deflection of the trajectory of the spacecraft (see Fig. 1), V_2 is the linear velocity of M_2 in its motion around the center of mass of the system M_1–M_2, μ_2 is the gravitational parameter of M_2, and ΔV is the increment of velocity obtained by the swingby. From those equations it is possible to get the fundamental well-known results:

1) The variation in energy ΔE is equal to the variation in angular momentum ΔC, for a system where $\omega = 1$. Otherwise, the correct expression is $\Delta E = \omega\Delta C$.

2) If the flyby is in front of the secondary body, there is a loss of energy. This loss has a maximum at $\psi = 90$ deg.

3) If the flyby is behind the secondary body, there is a gain of energy. This gain has a maximum at $\psi = 270$ deg.

There are many publications studying the standard swingby maneuver in different missions. Some examples are the study of missions to the satellites of the giant planets,[6–8] new missions for Pluto[9] and Neptune,[10] the study of the Earth's environment,[11–16] etc. There are also studies of the swingby maneuver under the model of the planar restricted three-body problem, such as Refs. 17–20.

Powered Swingby Maneuver

The description of the powered swingby is the main objective of this paper. The literature presents some interesting applications of this maneuver, such as an Earth–Mars mission using a swingby in Venus.[21] For the present research it is assumed that the difference between this maneuver and the standard one is that it is possible to apply an impulse to the spacecraft in the moment of the closest approach between the spacecraft and the secondary body. This impulse is allowed to have any magnitude, and it can have any direction that belongs to the plane of motion of the three bodies involved. Figure 2 shows the geometry of this maneuver and defines some of the variables used.

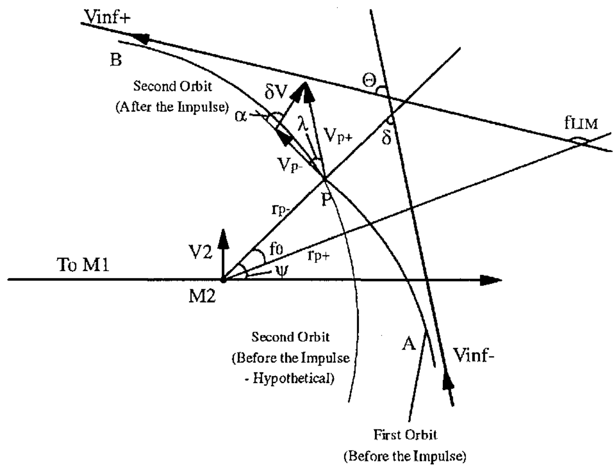


Fig. 2 Geometry of the powered swingby.

The variables are 1) $V_{\text{inf}-}$, the magnitude of the velocity of the spacecraft when approaching the celestial body; 2) V_{p-}, the magnitude of the velocity of the spacecraft at periapse before the impulse is applied; 3) V_{p+}, the magnitude of the velocity of the spacecraft at periapse after the impulse is applied; 4) δV, the magnitude of the impulse applied; 5) α, the angle between $\boldsymbol{V}_{p-}$ and the impulse applied [this variable defines the direction of the impulse; the range for α is $-180 < \alpha < 180$ deg (positive values are measured in the clockwise direction)]; 6) λ, the angle between $\boldsymbol{V}_{p-}$ and $\boldsymbol{V}_{p+}$; 7) r_{p-}, the distance between the spacecraft and the celestial body during the closest approach, when the spacecraft is in its first orbit (before the impulse); 8) r_{p+}, the distance between the spacecraft and the celestial body during the closest approach, when the spacecraft is in its second orbit (after the impulse); remember that the impulse changes the orbit of the spacecraft, and so there are two periapses involved in this maneuver: one that belongs to the first orbit (before the impulse) and one that belongs to the second orbit (after the impulse); 9) $V_{\text{inf}+}$, the magnitude of the velocity of the spacecraft when leaving the celestial body; and 10) ψ, the angle of approach (the angle between the periapse line and the line that connects the two primaries).

With those variables, it is possible to develop an algorithm to evaluate all of the parameters involved in this maneuver. The steps are shown next. Remember that the initial conditions (given variables) are $V_{\text{inf}-}$, r_{p-}, ψ for the geometry of the close approach, and δV and α to specify the impulse applied.

Step 1: Using the conservation of the energy it is possible to calculate V_{p-} from $V_{\text{inf}-}$ and r_{p-}. The equation used is $V_{p-} = \sqrt{[V_{\text{inf}-}^2 + (2\mu_2/r_{p-})]}$, where μ_2 is the gravitational parameter of the secondary body.

Step 2: Calculate V_{p+} from V_{p-}, δV, and α. The equation used is $V_{p+} = \sqrt{[V_{p-}^2 + \delta V^2 + 2V_{p-}\delta V \cos\alpha]}$.

Step 3: Then it is necessary to calculate $V_{\text{inf}+}$ from V_{p+} and r_{p-}. The equation is $V_{\text{inf}+} = \sqrt{[V_{p+}^2 - (2\mu_2/r_{p-})]}$.

Step 4: The next quantity to be evaluated is the semimajor axis a of the orbit after the swingby. It is obtained from $V_{\text{inf}+}$ by the use of the equation $a = (\mu_2/V_{\text{inf}+}^2)$.

Step 5: Then the quantity λ is calculated. It comes from V_{p-}, V_{p+}, and δV. The equation is $\lambda = \arccos[(\delta V^2 - V_{p-}^2 - V_{p+}^2)/-2V_{p-}V_{p+}]$.

Step 6: Then the angular momentum h, the semilactus rectum p, and the eccentricity e of the orbit after the swingby are calculated. They come from V_{p+}, r_{p-}, and λ. The equations are $h = r_{p-}V_{p+}\sin(90 - \lambda)$, $p = h^2/\mu_2$, and $e = \sqrt{[1 + (p/a)]}$.

Step 7: Calculate the true anomaly f_0 of the spacecraft in the second hyperbolic orbit (the orbit after the impulse) around the secondary body just after the impulse. It comes from e, p, and r_{p-}. The equation is $f_0 = \pm\arccos\{(1/e)[(p/r_{p-}) - 1]\}$. The rule to decide the sign of f_0 is the following: if $0 < \alpha < 180$ deg, the radial velocity gained a positive component from the impulse, which means that the spacecraft is escaping from the second body, and so it has already passed by the periapse and its true anomaly is positive; if

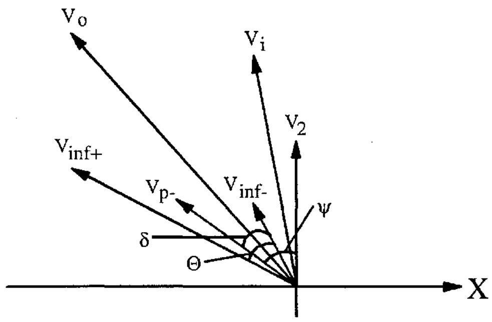


Fig. 3 Vector addition for the velocities.

$-180 < \alpha < 0$ deg, the radial velocity gained a negative component from the impulse, which means that the spacecraft is approaching the second body, and so it is still going to the periapse and its true anomaly is negative.

Step 8: Next calculate the true anomaly of the asymptotes of the second hyperbolic orbit of the spacecraft (f_{LIM}, the angle between the asymptote and the direction of the periapse; see Fig. 2), around the secondary body after the impulse. It comes from e. The equation is $f_{\mathrm{LIM}} = \arccos(-1/e)$.

Step 9: Then the total deflection for this maneuver is given by $\Theta = \delta - f_0 + f_{\mathrm{LIM}} - 90$ deg, where δ is the deflection angle before the impulse, f_0 is the deflection angle from the impulse until the passage by the periapse of the second hyperbolic orbit, and ($f_{\mathrm{LIM}} - 90$ deg) is the deflection angle from this passage by the periapse until the escape from the second body. Figure 2 shows those quantities.

Now it is necessary to continue the calculations to obtain the equations for the variation of energy, velocity, and angular momentum. Figure 3 shows the geometry of the vector addition, which provides the basic information to derive those equations.

From that figure it is possible to obtain the analytical equations required. The horizontal and the vertical components of the velocity before the close encounter (V_{ix} and V_{iy}) and after the close encounter (V_{ox} and V_{oy}) are

$$V_{ix} = -V_{\mathrm{inf-}} \sin(\psi - \delta) \tag{5}$$

$$V_{iy} = V_2 + V_{\mathrm{inf-}} \cos(\psi - \delta) \tag{6}$$

$$V_{ox} = -V_{\mathrm{inf+}} \sin(\psi - \delta + \Theta) \tag{7}$$

$$V_{oy} = V_2 + V_{\mathrm{inf+}} \cos(\psi - \delta + \Theta) \tag{8}$$

With those equations it is easy to calculate the variations in velocity, energy, and angular momentum. To derive those equations it is assumed that the impulse and the swingby maneuver are instantaneous and that the position of the spacecraft remains constant during the maneuver. The equations are

$$\Delta V_{\mathrm{imp}} = \sqrt{(V_{ox} - V_{ix})^2 + (V_{oy} - V_{iy})^2} \tag{9}$$

$$\Delta E_{\mathrm{imp}} = \tfrac{1}{2}\left(V_{ox}^2 + V_{oy}^2 - V_{ix}^2 - V_{iy}^2\right) \tag{10}$$

$$\Delta C_{\mathrm{imp}} = d(V_{oy} - V_{iy}) \tag{11}$$

where d is the distance between M_1 and M_2.

Results for the Two-Body Model

In this section the methods explained in the previous sections are used to generate some results to understand better this maneuver. The Earth–moon system is used as an example. A spacecraft makes a powered swingby with the moon for several values of the impulse (they can have different magnitudes and directions, but they are always in the plane of the motion of the three bodies). Figure 4 shows the variations in velocity, energy, and angular momentum for the powered swingby. The horizontal axis represents the angle α (in degrees) that defines the direction of the impulse, and the vertical axis represents the magnitude of the impulse (in kilometers per second). The parameters used for this maneuver are $\mu_1 = 398{,}600$ km^3/s^2,

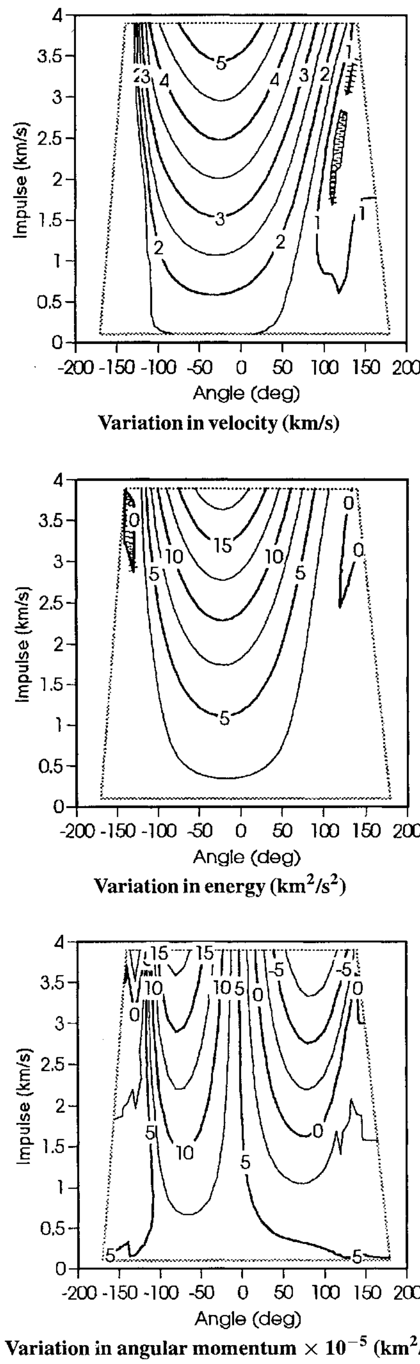


Fig. 4 Variation in velocity, energy, and angular momentum for powered swingby (ψ = 270 deg).

$\mu_2 = 4900$ km^3/s^2, $V_{\mathrm{inf-}} = 1.0$ km/s, $\psi = 270$ deg, $r_{p-} = 1900$ km, $V_2 = 1.02$ km/s, and $d = 384{,}400$ km. This maneuver generates an increase in the energy ($180 < \psi < 360$ deg). For the intervals $\alpha > 90$ and < -90 deg the impulse has a component opposite to the direction of motion of the spacecraft, decreasing the energy, and it is working against the swingby. The blank parts of the graphics ($\alpha > \approx 150$ and $< \approx -150$ deg) correspond to regions where the impulse caused the capture of the spacecraft by the moon. From Fig. 4 one can verify that the maximum transfer of velocity and energy occurs close to $\alpha = -20$ deg and that the minimum occurs close to the borders of the graphic. We can see this by following a line of constant ΔE (or ΔC). The minimum impulse to accomplish a given ΔE (or ΔC) occurs close to $\alpha = -20$ deg. Note also that there is a symmetry with respect to the line $\alpha \cong -20$ deg. This result is a little bit surprising in a first look into the problem. The expected result is to have the maximum at $\alpha = 0$ deg, to get the maximum transfer of energy from the impulse. The observed maximum oscillates around the value $\alpha = -20$ deg. The value for this observed maximum is up to 5% larger than the value for the expected maximum (case $\alpha = 0$ deg). The reason for that is the increase of the deflection angle after the impulse for $\alpha < 0$ deg. An impulse with a negative α gives a negative component to the radial velocity, which implies a negative value for f_0 (see step 7 discussed earlier). Then the total turning angle Θ will increase and it will also increase the total ΔE (for $\alpha = 0$ deg, f_0 would be zero). The extra gain in energy and velocity obtained from this increase in the turning angle compensates for the loss of energy transfer as a result of the nonalignment of the impulse with the velocity vector. The exact location of the optimum point depends on the particular case studied.

The graphic for the variation in angular momentum shows a different pattern, with symmetries with respect to the lines $\alpha \cong -90$ and $\cong 90$ deg. Remember that the maximum transfer of energy is not always the goal of the mission. A very close approach may be required to get data from the celestial body, but the consequent large increase in velocity and energy may not be desired for the continuation of the mission. In that way, a chart like that can provide important information for the mission designers, who can choose the parameters of the impulse that better satisfy the goals of the mission.

Next, Fig. 5 shows a similar maneuver, but with $\psi = 90$ deg, that is the case where the energy decreases in the standard swingby ($0 < \psi < 180$ deg). For the interval $\alpha > 90$ and < -90 deg the impulse has a component opposite to the direction of motion of the spacecraft, decreasing the energy, and it is working in favor of the swingby. For this maneuver there are positive and negative values for the change in energy. In the positive regions of the plot of the variation in energy, the impulse is dominating the swingby, and the net result is an increase in energy. In the negative regions of the same plot, the swingby and the impulse are working together to decrease the energy of the spacecraft. Note that positive values occur only above a certain limit in the magnitude of the impulse and that this limit decreases when α approaches zero. The variations in velocity and angular momentum present a behavior similar to the case of $\psi = 270$ deg. The same characteristic of maximum transfer of energy close to $\alpha = -20$ deg occurs here, besause of the same reason explained before.

After those first calculations, the efficiency of the powered maneuver is studied. The powered maneuver with maximum transfer of energy during the impulse ($\alpha = 0$ deg for a maneuver that increases the energy and $\alpha = 180$ deg for a maneuver that decreases the energy) is compared with a maneuver where the impulse is applied after the swingby. In this case the impulse is saved to be used when the spacecraft is out of the sphere of influence of the secondary body. The maneuver for $\alpha = 0$ deg is chosen for comparison to simplify the problem, since the real maximum changes from point to point. The differences are always less than 5%, which means that this simplification is acceptable. This new maneuver has two main steps: 1) a standard nonpropelled swingby with the same parameters of the powered maneuver (the same $V_{\text{inf}-}$, r_p, and ψ); and 2) then, in a second step, an impulse (with the same magnitude δV of the impulse used in the powered swingby) that is applied after the spacecraft leaves the secondary body. This impulse is assumed to be applied in a direction that extremizes the transfer of energy. It means that for the maneuvers where the goal is to increase the energy ($180 < \psi < 360$ deg) this impulse is posigrade (applied in the direction of the motion of the spacecraft) and for the maneuvers where the goal is to decrease the energy ($0 < \psi < 180$ deg) this impulse is retrograde (applied in the direction opposite to the motion of the spacecraft). Figure 6 shows the results for $\psi = 90$ and 270 deg and for $V_{\text{inf}-} = 1.0$ and 2.0 km/s. The quantity plotted is $|\Delta E_{\text{imp}}| - |\Delta E_{\text{impafter}}|$ (in square kilometers per square second), where ΔE_{imp} is the energy variation obtained by the powered swingby, and $\Delta E_{\text{impafter}}$ is the energy variation of the maneuver that applies the impulse after the close approach. The system of axis has r_p (distance of closest approach, in kilometers) in the horizontal axis and ψ (the angle of approach, in degrees) in the vertical axis.

It means that a positive value for this quantity indicates that the application of the impulse during the close approach is more efficient (in terms of causing a variation in energy of larger magnitude) than

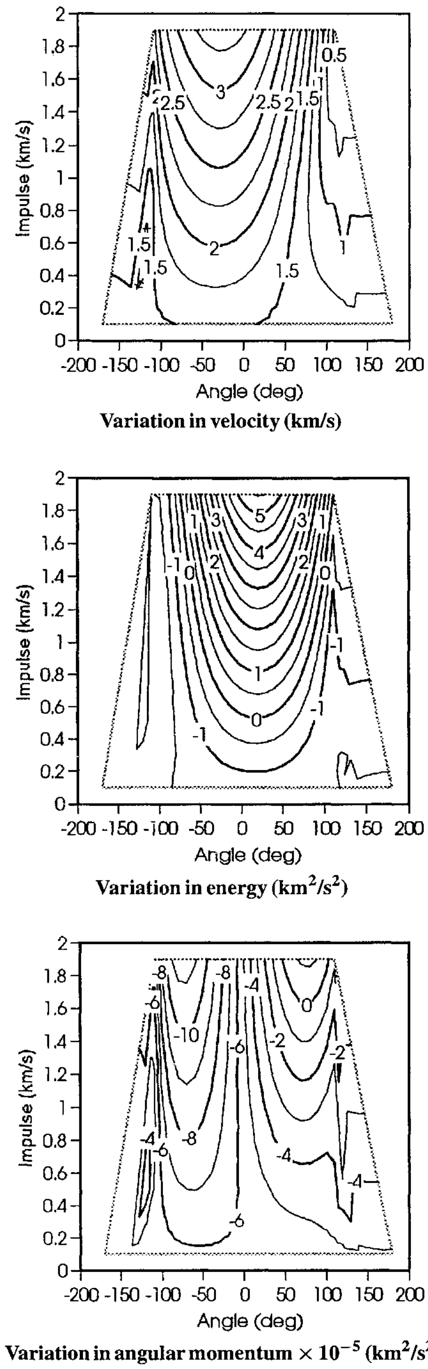


Fig. 5 Variation in velocity, energy, and angular momentum for powered swingby (ψ = 90 deg).

the application of an impulse with the same magnitude after the close approach. The precise definition of efficiency is the quantity of extra energy that is obtained when the impulse is applied during the periapse passage, when compared with the maneuver that applied the impulse after the swingby, for an impulse with fixed magnitude.

To obtain the numerical value for the $\Delta E_{\text{impafter}}$ it is necessary to follow the steps shown next.

Step 1: Evaluate the energy before the close approach E_i from the equation $E_i = \frac{1}{2}(V_{ix}^2 + V_{iy}^2) - (\mu_1/d)$, where μ_1 is the gravitational parameter of the primary body and d is the distance M_1–M_2.

Step 2: Next, the energy after the standard swingby maneuver is obtained directly from the expression $E_o = E_i - 2V_2 V_{\text{inf}-} \sin\delta \sin\psi$.

Step 3: Then the magnitude of the velocity after the standard swingby maneuver is calculated from the energy, using the expression $V_o = \sqrt{2}[E_o + (\mu_1/d)]$.

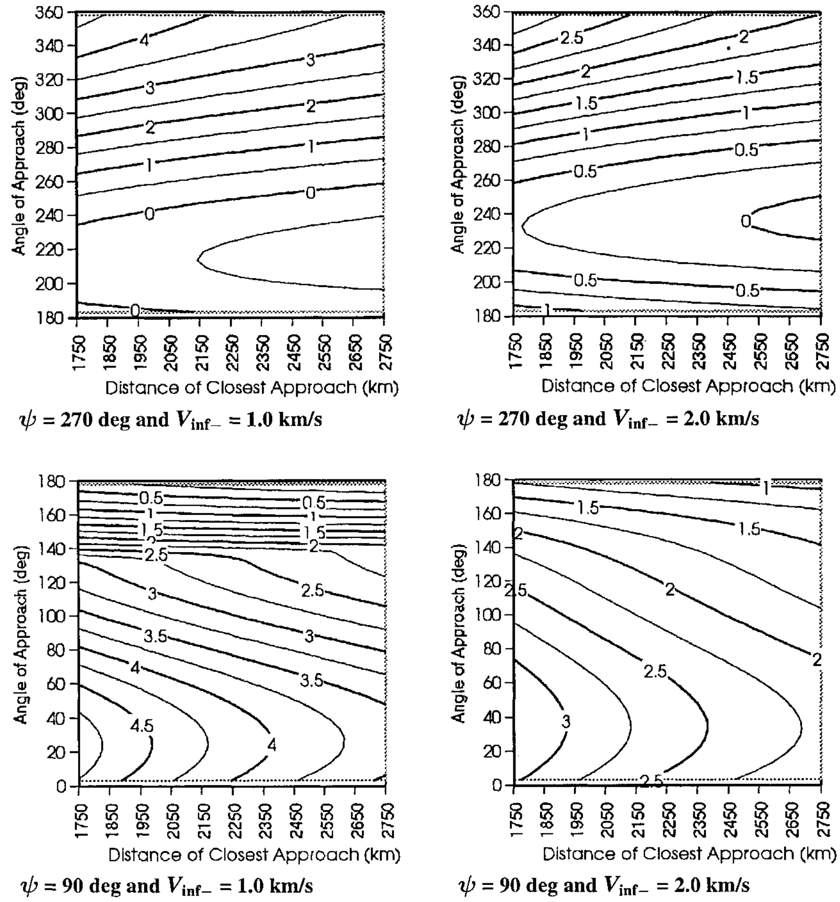


Fig. 6 Efficiency of the powered swingby.

Step 4: Finally, the $\Delta E_{\rm impafter}$ is obtained from $\Delta E_{\rm impafter} = 1/2\ (V_o + \delta V)^2 - (\mu_1/d) - E_i$, if $180 < \psi < 360$ deg; or $\Delta E_{\rm impafter} = 1/2\ (V_o - \delta V)^2 - (\mu_1/d) - E_i$, if $0 < \psi < 180$ deg.

It is clear that the efficiency is highly dependent on the angle of approach and that it has little dependence on the distance of closest approach. We also see that the efficiency has the same behavior for the cases $V_{\rm inf-} = 1.0$ and 2.0 km/s. There is only a shift in the values. It is also clear that there is a relatively small area of negative values, which means that an impulsive swingby maneuver is a better choice for most of the cases, but there are exceptions. Note that the plots match very well at $\psi = 180$ deg, although the goal of the maneuver (gain or lose of energy) changes at this point.

Results Using the Restricted Three-Body Problem

The goal of this section is to reproduce the maneuvers calculated in the previous sections using the well-known planar circular restricted three-body problem as the dynamical model. This model is a much better approximation of the reality, because it takes into account the gravitational forces of the two primaries during all of the phases of the mission. There is no need to neglect some of those forces during some steps. The forces not modeled by this model are really very small (like the gravitational forces from distant bodies, radiation pressure, etc.), and so very small differences are expected between this model and the exact solution.

This model assumes that two main bodies (M_1 and M_2) are orbiting their common center of mass in circular Keplerian orbits and a third body (M_3), with negligible mass, is orbiting these two primaries. The motion of M_3 is supposed to stay in the plane of the motion of M_1 and M_2, and it is affected by both primaries, but it does not affect their motion.[22] The canonical system of units is used, and it implies that 1) the unit of distance is the distance between M_1 and M_2; 2) the angular velocity ω of the motion of M_1 and M_2 is assumed to be 1; 3) the mass of the smaller primary M_2 is given by $\mu = m_2/(m_1 + m_2)$ (where m_1 and m_2 are the real masses of M_1 and M_2, respectively), and the mass of M_1 is $(1 - \mu)$, so that the total mass of the system is 1; 4) the unit of time is defined such that the period of the motion of the primaries is 2π; and 5) the gravitational constant is 1.

Then the equations of motion in the rotating frame are

$$\ddot{x} - 2\dot{y} = x - \frac{\partial V}{\partial x} = \frac{\partial \Omega}{\partial x} \tag{12}$$

$$\ddot{y} + 2\dot{x} = y - \frac{\partial V}{\partial y} = \frac{\partial \Omega}{\partial y} \tag{13}$$

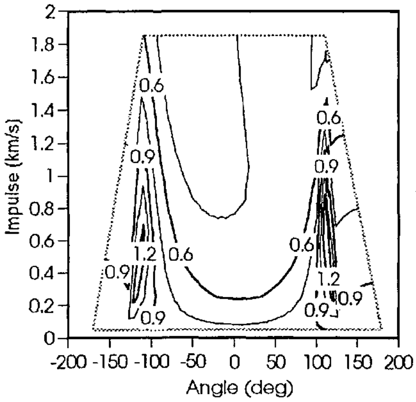
where Ω is the pseudopotential given by

$$\Omega = \tfrac{1}{2}(\dot{x}^2 + \dot{y}^2) + [(1-\mu)/r_1] + (\mu/r_2) \tag{14}$$

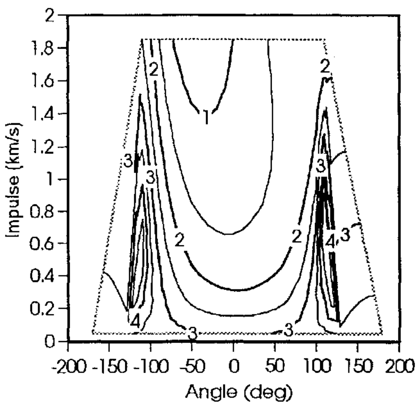
This system of equations has no analytical solutions, and numerical integration is required to solve the problem.

The equations of motion given by Eqs. (12–14) are right, but they are not suitable for numerical integration in trajectories passing near one of the primaries. The reason is that the positions of both primaries are singularities in the potential V (since r_1 or r_2 goes to zero, or near zero) and the accuracy of the numerical integration is affected every time this situation occurs.

The solution for this problem is the use of regularization, which consists of a substitution of the variables for position (x–y) and time (t) by another set of variables (ω_1, ω_2, and τ), such that the singularities are eliminated in these new variables. Several transformations with this goal are available in the literature, such as Thiele-Burrau, Lemaître, and Birkhoff (see Ref. 22, Chap. 3). They are called global regularization, to emphasize that both singularities are eliminated at the same time. The case where only one singularity is eliminated at a time is called local regularization. For the present research Lemaître's regularization is used. More details are available in Ref. 22.



Energy (km²/s²)



Angular momemtum (km²/s)

Fig. 7 Comparison between the two models used (two and three bodies).

Figure 7 shows the difference between the results obtained using the two-body celestial mechanics and the new results obtained using the restricted three-body problem for the maneuver with $\psi = 90$ deg. Results are similar for other cases studied and are not shown here to save space. The quantities shown are defined as (value for the two-body model) − (value for the three-body model). To make the plots more clear, the results for the energy are multiplied by 10, and the results for the angular momentum are divided by 10^4. The magnitudes of the differences go from very close to zero until 0.15 (in energy) and 4×10^4 (in angular momentum). Those numbers represent maximum errors on the order of a few percentages (less than 10) in both cases. The errors are smaller in the interval $-90 < \alpha < 90$ deg, and they grow up close to the border of the graphics. It means that the two-body approximation of this maneuver gives better results when $-90 < \alpha < 90$ deg. It is also possible to conclude that this approximation increases in quality when the magnitude of the impulse increases.

Conclusions

A method to calculate the variations in velocity, energy, and angular momentum for the powered swingby is developed based on the patched-conic approximation. Numerical examples are calculated for the Earth–moon system to test and validate the algorithm. An interesting result that was found is that the best direction to apply an impulse is not the direction of motion of the spacecraft but a direction that makes an angle between close to −20 deg with the motion of the spacecraft (a negative angle means that the impulse has a radial component pointing to the secondary body), as a result of the increase of the turning angle in this maneuver. Then the powered swingby maneuver is compared with a different maneuver that is performed in two steps: 1) a nonpropelled swingby and 2) an impulsive thrust applied after the swingby. In that way, it is possible to investigate the best position to apply the impulse. The results show that for the majority of the cases studied the powered swingby is a better choice, but there are some cases where the unpowered swingby followed by the impulse increases the variation in energy obtained for an impulse with a fixed magnitude. Next, the maneuvers are reproduced under the dynamical model given by the restricted three-body problem. The differences between the results are shown. It is possible to conclude that the two-body problem gives a better approximation in the interval $-90 < \alpha < 90$ deg and that this approximation increases in quality when the magnitude of the impulse increases.

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